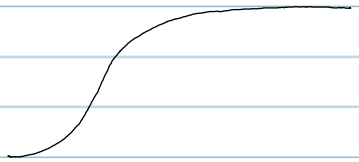
Section 2.2 Exponential Functions

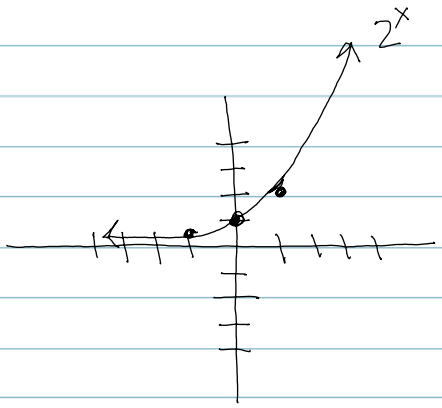
$y = b^x$



2.4 → Final

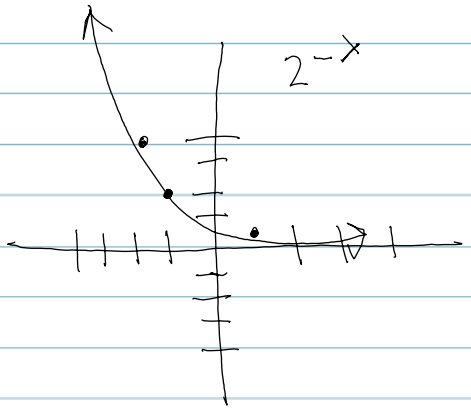
$y = 2^x$

x	y
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

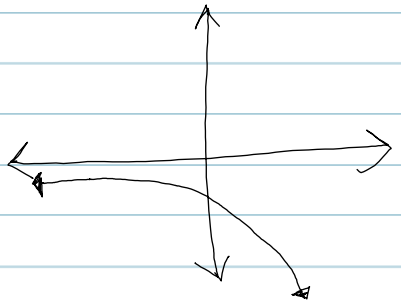


$y = 2^{-x}$

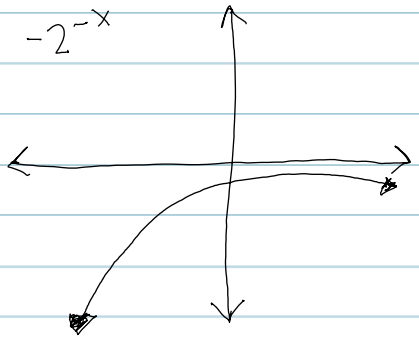
x	y
-2	$2^2 = 4$
-1	$2^{-1} = 2^1 = 2$
0	$2^{-0} = 1$
1	$2^{-1} = \frac{1}{2}$
2	$2^{-2} = \frac{1}{4}$

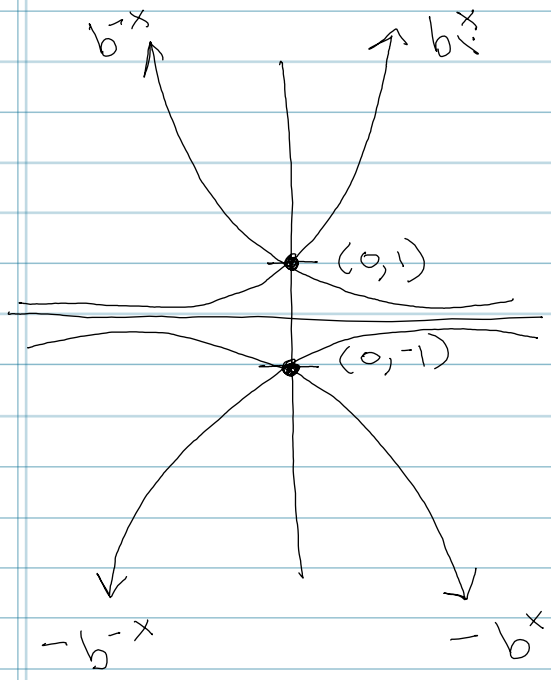


$y = -2^x$



$y = -2^{-x}$





$$b^{x-h}+k$$

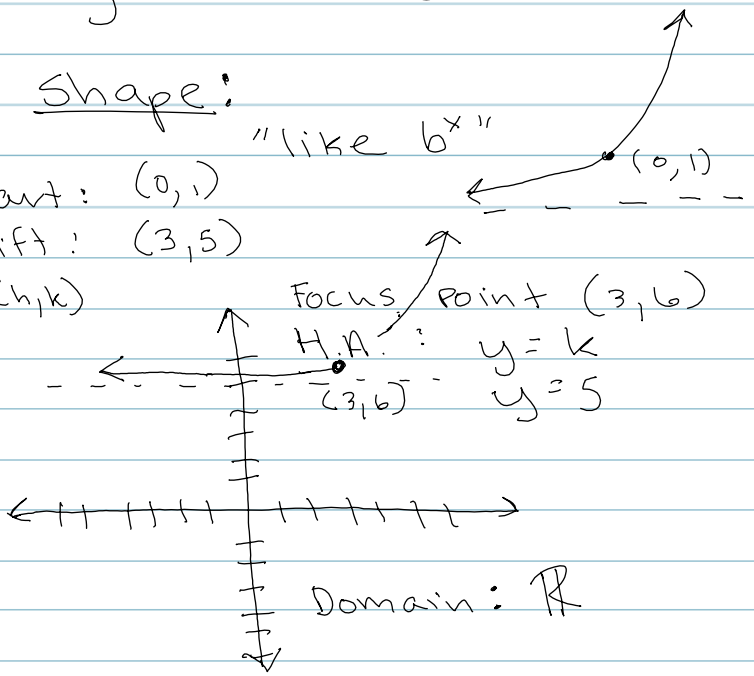
$$y=2^{x-3}+5$$

Shape: "like b^x"

start: (0,1)

shift: (3,5)

(h,k)



Range: $y>5$

$(5,\infty)$

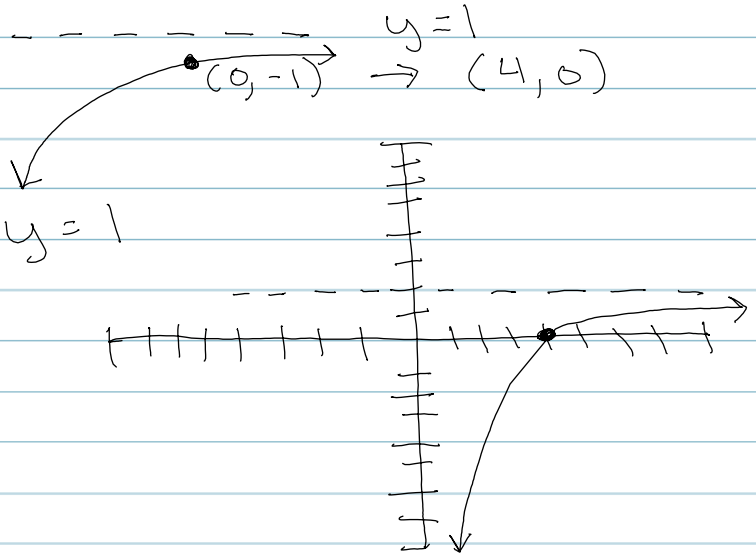
$$y=-3^{4-x}+1$$

① Shape: $-b^{-x}$

② Start: (0,-1)

③ Shift: (4,1) / (4,0)

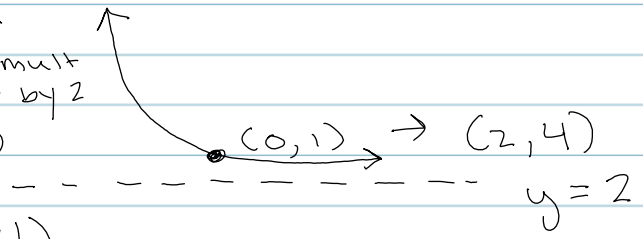
④ H.A. $y=k$ $y=1$



$y = 2 \cdot 3^{2-x} + 2$

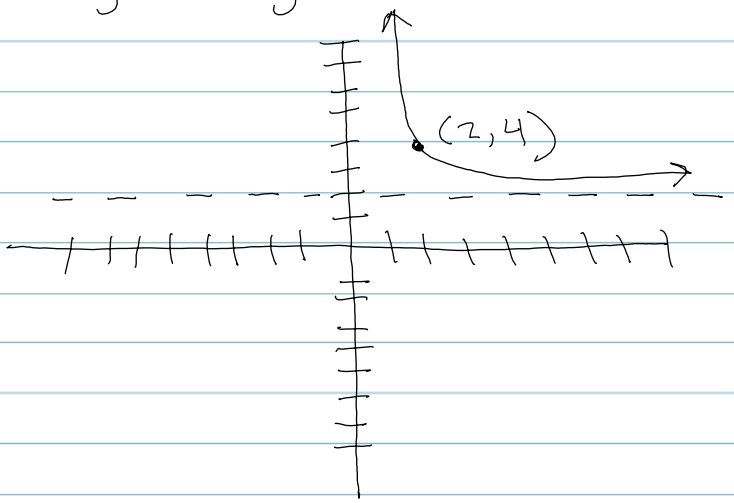
① Like: b^{-x}

② Start: $(0,1)$ mult by 2 $= (0,2)$



③ Shift: $(2,4)$

④ H.A.: $y = k$ $y = 2$



Domain: $\mathbb{R}$

Range: $(2, \infty)$

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = accumulated amount

agree { r = rate, t = time

n = # of compounding periods

n = 1 yearly

n = 2 bi-annually

n = 4 quarterly

n = 12 monthly

n = 360 - daily
365 -

Compounded continuously

$A = Pe^{rt}$ $\quad e \cong 2.71\ldots$

Example

$P = \$10,000$

$r = 8\%$

$t = 20, 30, 40, 50$ years

$n = 4$ quarterly

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 10,000\left(\frac{0.08}{4}\right)^{4t}$

$A = 10,000\,(1.02)^{4t}$

$t = 26$	48,754
$= 30$	107,651
$= 40$	237,699
$= 50$	524,848

Example

Invest $\$2,000$ at 10% APR, compounded continuously

$A = Pe^{rt}$

$A = (2000)\,e^{0.10(5)} = 2000e^{0.50}$

$= 2000e^{x}\ 0.50$ (2nd)

$= \$3,297.44$

General Growth & Decay

$y = A \cdot b^x$

Section 2.2

(80) $t = 0$ (hrs) $\quad y = 1000$ bacteria

$t = 2 \quad y = 1500$

$y = A \cdot b^x$

$1000 = A \cdot b^0$

$1000 = A$

$y = 1000b^x$

$1500 = 1000b^2$

$\frac{1500}{1000} = 1.5 = b^2$

$\pm\sqrt{1.5} = b$

$1.2247 = b$

$y = 1000(1.2247)^x$

$x = 48$ hrs $\quad y = 1000(1.2247)^{48} = 16,804,533$ bacteria two days later.

(20)

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{2}$	1	2	4	8
$g(x)$	3	0	-1	0	3

(each step $\cdot 2$)

a $(0,2)$ $\times 2$

$y = 2 \cdot 2^x$

Parabolic. Not exponential.

(56)

$(2, -4)$ $\quad y = Ab^x$

$(4, -16)$ $\quad -4 = A \cdot b^2$ $\quad -16 = A \cdot b^4$

$-4 = A \cdot b^2$

$y = A \cdot 2^x$ $\quad (x, y)$ $\quad (2, -4)$ $\quad 4 = b^2$

$-4 = A \cdot 2^2$ $\quad 2 = b$

$-4 = A \cdot 4$

$-1 = A$ $\quad y = -2^x$